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Numerical Analysis of Stress Intensity Factors of a Crack in the Composite Patch Repair

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ABSTRACT

The composite patch repair technology can be utilized to provide upgrades, such as higher design requirements and life extensions. However, in this case, failures easily happen on the edge interface between the substrate material and the restorative material, because of the incompatible of different properties. Failures such as flaws or cracks lying along the interface reduce the strength of the structure significantly. In this paper, the numerical solutions of hypersingular integro-differential equations are discussed in the analysis of three dimensional interfacial cracks subjected to general internal pressure, and the problem is formulated on the basis of the body force method. The stress intensity factors are given with varying material combinations. It is found that the stress intensity factors and are determined by the bi-materials constant alone, independent of elastic modulus ratio and Poisson's ratio.

KEY WORDS: Composite patch repair; hypersingualr intergro-differential equations; body force method; stress intensity factors.

INTRODUCTION

In recent years, composite materials are being used in wide range of marine engineering such as ships, platforms, pipelines, FPSO and FSO. On these marine engineering structural defects such as corrosion damage and cracks are typically repaired by welding. However, welding is unwanted hot work on these structures because it requires shutdown of parts of them thus resulting in expensive production delays (Weizenbock and McGeorge 2012). Recently, composite patch repair are popular used in the marine engineering repair, which is called the cold repair (see Fig.1).

Composite patch repair, effected using strong and stiff fiber composite materials, have been used for several decades in the defense industry for the maintenance and life extension of aircraft and warships (Baker, Kelly and Dutton 2004), but more recently they have also been used in the marine, infrastructure and even the oil and gas industries (Baker, Kelly and Dutton SE 2004; Frost 2005; Marsh 2006). In the oil and gas industry, composite repairs can often be seen in the maintenance of aged pipework (Frost 2005; Marsh 2006; Hawkins, Grabow, Walker and Keller 2012; Hill, Ertekin, Sridhar and Scott 2009; Popineau, Wiet, Bouletd and Auria 2012). Composite patch repair solutions have many advantages over conventional repairs to marine structures such as avoidance of hot work （welding）and the ability to seal cracks. Apart from this, composite repairs are also attractive because they are adaptable to virtually any substrate geometry, easily conforming to complex shapes and fitting into tight spots. In addition, the anisotropy of composite materials also affords design flexibility that contributes to cost and property optimization. Their light weight means that heavy lifting machinery is not necessary and operators can generally work with the materials with improved safety. Finally, with a wide choice of materials available, i.e., fiber types, reinforcement forms and resins, and processes, to fit various operating conditions and environments, it is no wonder that composite repairs have been receiving much attention from the industry.

Generally, the possible failure mechanisms in a bonded composite repair are shown schematically in Fig. 2. Failure can occur in the repair (i.e., through sustained external damage), in the interface between the repair and the steel substrate, including the adhesive layer (i.e., through peeling and disbond), and also in the steel substrate (i.e., through continued deterioration of the defect in the metal substrate). The designer is required to consider all these possibilities and to define acceptance criteria for each of the critical failure mechanism (McGeorge, Echtermeyer, Leong, Melve, Robinson e, and Fischer 2009) .

(a)

Figure 1 Composite patch repair of the corroded deck floor of the FSO ABU Cluster ((a) shows the deck before the repair), (b) shows the deck after the repair)

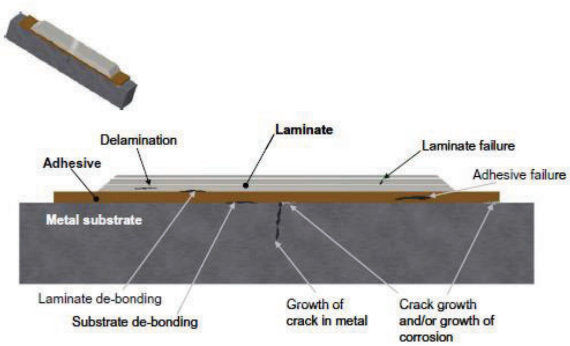

Figure 2 Potential failures of a bonded composite repair.

In this paper, a small rectangular crack is proposed on the interface edge of bonded composite repair, as the crack size is very small compared to the size of substrate, which could be treated as the infinite plate comparing with the crack. Therefore, the simulation model could be illustrated in Fig.3, where a rectangular crack is on the interface of two bonded dissimilar elastic half-infinite spaces.

For a new repair method, an important topic is the lifetime. Today, no established accelerated test method exists that can reliably predict the lifetime of bonded repair. In the elastic fracture mechanics, the parameters of stress intensity factors of a crack are often used to provide the basis of predicting the strength and lifetime. This paper discussed the stress intensity factors of the crack (shown in Fig.3) on the bonded repair based on the numerical analysis.

NUMERICAL ANALYSIS

Body force method is an important numerical method to solve the crack problem. Nistani (1974) firstly proposed the method, and the singular integral equations are constructed for the crack problem. Its numerical solutions were investigated in the continued studies (Noda and Oda 1992; Noda and Matsuo 1998). Noda and Oda (1997) discussed the stress intensity factors of cracks considering the interaction of them, and Noda and Xu(2007) discussed the stress intensity factor of a rectangular crack under the constant tension loading using body force method. In this paper, using body force method, the stress intensity factors of a rectangular crack on the interface of bonded half-infinite plates (see Fig.3) under general pressure are discussed.

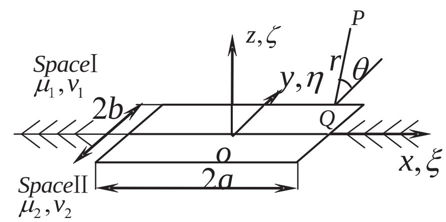

Fig.3. Interfacial rectangular crack subjected to general internal pressure $p_z(x,y)$.

Hypersingular integro-differential equations

Supposing the two elastic half-spaces are bonded together along the x–y plane with a fixed rectangular Cartesian coordinate system x_i ($i = x, y, z$), and the upper half-space is occupied by an elastic medium with constants $(\mu_1,\ \nu_1)$, and the lower half-space by an elastic medium with constants $(\mu_2,\ \nu_2)$ (see Fig.3). Here, μ_1 , μ_2 are shear modulus for space 1 and space 2, and ν_1, ν_2 are Poisson's ratio for space 1 and space 2. The crack is assumed to be located at the bi-material interface.

Hypersingular intergro-differential equations for two dimensional cracks when $a/b \to \infty$ (see Fig.3) on a bi-material interface are shown in Eq. 1 (Noda, Kagita, Chen and Oda 2003) .

$$\beta C \frac{\partial \Delta u_z}{\partial y} + \frac{C}{\pi} \int_{-b}^{b} \frac{\Delta u_y}{(y-\xi)^2} d\xi = -p_y \tag{1a}$$

$$-\beta C \frac{\partial \Delta u_y}{\partial y} + \frac{C}{\pi} \int_{-b}^{b} \frac{\Delta u_z}{(y-\xi)^2} d\xi = -p_z$$

$$C = \frac{2\mu_1(1+\alpha)}{(1-\beta^2)(\kappa_1+1)} = \frac{2\mu_2(1-\alpha)}{(1-\beta^2)(\kappa_2+1)}$$

$$\varepsilon = \frac{1}{2\pi} \ln[(\frac{\kappa_1}{\mu_1} + \frac{1}{\mu_2}) / (\frac{\kappa_2}{\mu_2} + \frac{1}{\mu_1})] \tag{1b}$$

$$\kappa_m = \begin{cases} (3-4\nu_m)/(1+\nu_m), & \text{Plane stress} \\ 3-4\nu_m, & \text{Plane strain} \end{cases} \quad m = 1,2 \tag{1c}$$

$$\Delta u_i(y) = u_i(y,0^+) - u_i(y,0^-)(i = y,z) \tag{1d}$$

Here Δu_x , Δu_y are the crack opening displacements.

Hypersingular intergro-differential equations for three dimensional cracks on a bi-material interface were derived by Chen–Noda-Tang (1999) and expressed as shown in Eq. 2 (Chen, Noda and Tang 1999).

$$\mu_1\left(\Lambda_2-\Lambda_1\right)\frac{\partial\Delta u_z\left(x,y\right)}{\partial x}+\mu_1\frac{\left(2\Lambda-\Lambda_1-\Lambda_2\right)}{2\pi}\times\int_s\frac{1}{r^3}\Delta u_x(\xi,\eta)dS(\xi,\eta)+3\mu_1\frac{\left(\Lambda_1+\Lambda_2-\Lambda\right)}{2\pi}\times\int_s\frac{\left(x-\xi\right)^2}{r^5}\Delta u_x(\xi,\eta)dS(\xi,\eta)+3\mu_1\frac{\left(\Lambda_1+\Lambda_2-\Lambda\right)}{2\pi}\int_s\frac{\left(x-\xi\right)\left(y-\eta\right)}{r^5}\times\Delta u_y(\xi,\eta)dS(\xi,\eta)=-p_x(x,y) \quad (2a)$$

$$\mu_1\left(\Lambda_2-\Lambda_1\right)\frac{\partial\Delta u_z\left(x,y\right)}{\partial y}+\mu_1\frac{\left(2\Lambda-\Lambda_1-\Lambda_2\right)}{2\pi}\times\int_s\frac{1}{r^3}\Delta u_y(\xi,\eta)dS(\xi,\eta)+3\mu_1\frac{\left(\Lambda_1+\Lambda_2-\Lambda\right)}{2\pi}\times\int_s\frac{\left(x-\xi\right)\left(y-\eta\right)}{r^5}\Delta u_x(\xi,\eta)dS(\xi,\eta)+3\mu_1\frac{\left(\Lambda_1+\Lambda_2-\Lambda\right)}{2\pi}\int_s\frac{\left(y-\eta\right)^2}{r^5}\times\Delta u_y(\xi,\eta)dS(\xi,\eta)=-p_y(x,y) \quad (2b)$$

$$\mu_1\left(\Lambda_1-\Lambda_2\right)\left(\frac{\partial\Delta u_x\left(x,y\right)}{\partial x}+\frac{\partial\Delta u_y\left(x,y\right)}{\partial y}\right)+\mu_1\frac{\left(\Lambda_1+\Lambda_2\right)}{2\pi}\int_s\frac{1}{r^3}\Delta u_z(\xi,\eta)dS(\xi,\eta)=-p_z\left(x,y\right) \quad (2c)$$

$$\Lambda=\frac{\mu_2}{\mu_1+\mu_2},\quad \Lambda_1=\frac{\mu_2}{\mu_1+\kappa_1\mu_2},\quad \Lambda_2=\frac{\mu_2}{\mu_2+\kappa_2\mu_1},$$
$$\kappa_1=3-4\nu_1,\quad \kappa_2=3-4\nu_2,\quad r^2=\left(x-\xi\right)^2+\left(y-\eta\right)^2 \quad (2d)$$
$$\left(x,y\right)\in S, S=\left\{\left(x,y\right)\|\,x|\le a,|\,y|\le b\right\}$$

$$\Delta u_i\left(x\right)=u_i(x,y,0^+)-u_i(x,y0^-)(i=x,y,z) \quad (2e)$$

In Eq.2, unknown functions are crack opening displacements $\Delta u_x\left(x,y\right),\Delta u_y\left(x,y\right),\Delta u_z\left(x,y\right)$, Here, (ξ,η,ζ) is a rectangular coordinate (x,y,z) where the displacement discontinuities are distributed, the notations p_x, p_y, p_z denote surface tractions in the x,y,z directions at the crack surface. Since the integral has a hypersingularity of the form r^{-3} when $x=\xi$ and $y=\eta$, the integration should be interpreted in a sense of a finite part integral in the region S.

Numerical solutions

In the present analysis, the fundamental densities and polynomials have been used to approximate the unknown functions as continuous functions. First, we put

$$\Delta u_x\left(\xi,\eta\right)=w_x(\xi,\eta)F_x(\xi,\eta),$$
$$\Delta u_y\left(\xi,\eta\right)=w_y(\xi,\eta)F_y(\xi,\eta), \quad (3)$$
$$\Delta u_z\left(\xi,\eta\right)=w_z(\xi,\eta)F_z(\xi,\eta).$$

The fundamental densities $w(\zeta,\eta)$ lead to express the oscillation stress singularity and overlapping of crack surfaces along the crack front exactly.

$$w_x(\xi,\eta)=\sum_{l=1}^{2}\frac{1+k_l}{4\mu_l\cosh\pi\varepsilon}\sqrt{a^2-\xi^2}\sqrt{b^2-\eta^2}\sin\left(\varepsilon\ln\left(\frac{a-\xi}{a+\xi}\right)\right),$$
$$w_y(\xi,\eta)=\sum_{l=1}^{2}\frac{1+k_l}{4\mu_l\cosh\pi\varepsilon}\sqrt{a^2-\xi^2}\sqrt{b^2-\eta^2}\sin\left(\varepsilon\ln\left(\frac{b-\eta}{b+\eta}\right)\right),$$
$$w_z(\xi,\eta)=\sum_{l=1}^{2}\frac{1+k_l}{4\mu_l\cosh\pi\varepsilon}\sqrt{a^2-\xi^2}\sqrt{b^2-\eta^2}\cos\left(\varepsilon\ln\left(\frac{a-\xi}{a+\xi}\right)\right)\times\cos\left(\varepsilon\ln\left(\frac{b-\eta}{b+\eta}\right)\right). \quad (4)$$

To satisfy the boundary conditions for the rectangle region of the interfacial crack, $F_i(\xi,\eta)(i=x,y,z)$ can be approximated by polynomials, for example:

$$F_x(\xi,\eta)=\alpha_0+\alpha_1\eta+\cdots+\alpha_{n-1}\eta^{(n-1)}+\alpha_n\eta^n+\alpha_{n+1}\xi+\alpha_{n+2}\xi\eta+\cdots+\alpha_{2n}\xi\eta^n+\cdots+\alpha_{l-n-1}\xi^m+\alpha_{l-n}\xi^m\eta+\cdots+\alpha_{l-1}\xi^m\eta^n=\sum_{i=0}^{l-1}\alpha_iG_i(\xi,\eta), \quad (5)$$
$$l=\left(m+1\right)\left(n+1\right),$$
$$G_0\left(\xi,\eta\right)=1,\ G_1\left(\xi,\eta\right)=\eta,\ \cdots,G_{n+1}\left(\xi,\eta\right)=\xi,\ \cdots,G_{l-1}\left(\xi,\eta\right)=\xi^m\eta^n.$$

RESULTS AND DISCUSSION

Stress intensity factors K_I, K_{II} and dimensionless stress intensity factors F_I, F_{II} are defined as the following:

$$K(Q)=K_I(Q)+iK_{II}(Q)=\lim_{r\to 0}\sqrt{2\pi r}^{1/2-i\varepsilon}[\sigma_z(r,\theta)+i\tau_{zx}(r,\theta)]_{\theta=0}$$

$$F_I+iF_{II}=\frac{K_I(x,y)\big|_{y=\pm b}+iK_{II}(x,y)\big|_{y=\pm b}}{\sigma_z^\infty\sqrt{\pi b}}=\sqrt{a^2-x^2}\left(\cos\left(\varepsilon\ln(\frac{a-x}{a+x})\right)F_z(x,y)\big|_{y=\pm b}+2i\varepsilon F_y(x,y)\big|_{y=\pm b}\right) \quad (6)$$

It is known that any kinds of expression loads could be expanded using Taylor expansion, so the dimensionless stress intensity factors of the rectangular crack under polynomial pressure is calculated using the above method, then the results for the crack under general loadings could be obtained.

Table 1 shows the dimensionless stress intensity factors F_I and F_{II} when $a/b\to\infty$ (a/b=8) for the crack under general internal pressure p_0, $p_0\left(y/a\right)^2$ and $p_0\left(y/a\right)^3$. These results are obtained by applying a similar numerical solution described above. For constant pressure $p_z\left(y\right)=p_0$, the exact solution is obtained at $M=1$, and for $p_z(y)=p_0\left(y/a\right)^2$, the exact solution is obtained at $M=3$. Generally, it could be concluded that for $p_z(y)=p_0\left(y/a\right)^n$, the exact solution is obtained at $M = n+1$. To clearly illustrate this conclusion, Figure 4 is made according to Table 1 except the case of $p_z(y) = p_0$, from the relationship between F_I, F_{II} and M, it could be found that the convergent results could be obtained whatever the polynomial index of the internal pressure is.

It is found that dimensionless stress intensity factors F_I and F_{II} are constant for the variation of the shear modulus ratio μ_2/μ_1 and Poisson's ratio ν_1, $\nu_2 = 0 \sim 0.5$ if ε is constant. Figure 5 shows the dimensionless stress intensity factors F_I and F_{II} at the point $(x, y) = (0, b)$ with different values ε in Fig.3. In other words, the stress intensity factors K_I and K_{II} of planer interface cracks in bi-materials are determined by the bi-material constant ε alone, independent of the shear modulus ratio and Poisson's ratio, and of course, Young's modulus ratio.

CONCLUSIONS

The model of a crack in the composite patch repair in the marine engineering is constructed in this paper, and the stress intensity factors are calculated from solving the hypersingular integro-differential equations on the basis of body force method, which is very important to evaluate the strength and lifetime of the cold repair method. The conclusions can be summarized as follows:

(1)Dimensionless stress intensity factors of a rectangular crack on the interface of two bonded half-infinite plates have convergent solutions, and they are obtained with varying material combinations when the crack is under polynomial internal pressure.

(2)The stress intensity factor is determined by the bi-material constant ε alone, independent of the shear modulus ratio and Poisson's ratio, and of course, Young's modulus ratio.

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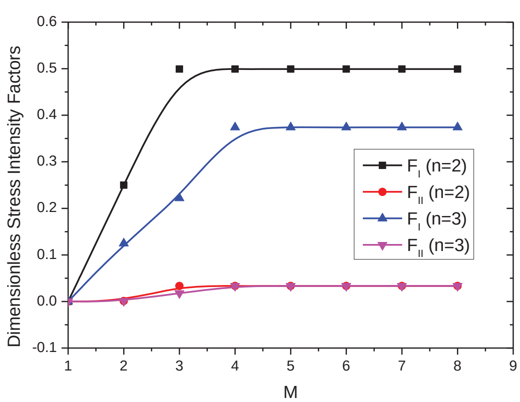
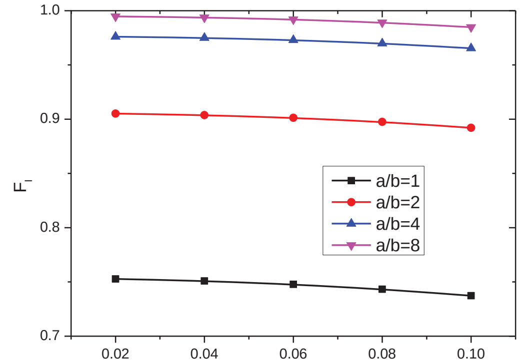
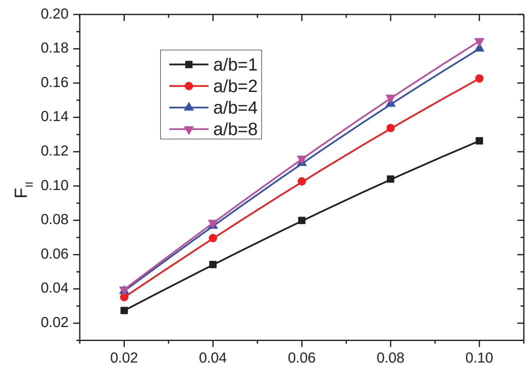


Figure 4 Dimensionless stress intensity factors F_I and F_{II} at $\varepsilon = 0.02$ when $a/b \to \infty$ with different M values .



(a)



(b)

Figure 5 Dimensionless stress intensity factors at the point $(x, y) = (0, b)$ in Fig. 3 under constant pressure p_0, (a) F_I; (b) F_{II}

Table 1 Dimensionless stress intensity factors and at when with different M values.

	$p_z(y) = p_0$		$p_z(y) = p_0(y/a)^2$		$p_z(y) = p_0(y/a)^3$	
M	F_I	F_{II}	F_I	F_{II}	F_I	F_{II}
1	1.0000	0.0400	0.0000	0.0000	0.0000	0.0000
2	1.0000	0.0400	0.2500	0.0010	0.1248	0.0010
3	1.0000	0.0400	0.4992	0.0333	0.2219	0.0178
4	1.0000	0.0400	0.4992	0.0333	0.3741	0.0333
5	1.0000	0.0400	0.4992	0.0333	0.3741	0.0333
6	1.0000	0.0400	0.4992	0.0333	0.3741	0.0333
7	1.0000	0.0400	0.4992	0.0333	0.3741	0.0333
8	1.0000	0.0400	0.4992	0.0333	0.3741	0.0333

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